

# Critical Sensitivity and Fluctuations in Catastrophic Rupture

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## Abstract

**Rupture of heterogeneous crust appears to be a sudden transition. Since its sensitivity to heterogeneity and stress transfer in different scales play a fundamental role in evolution, the threshold of earthquake-like rupture is uncertain. This seems to be the root of the difficulty of earthquake prediction. We currently found that critical sensitivity is a common feature near the transition to rupture in evolution induced catastrophe (EIC). Such a feature can be adopted as a precursor of catastrophic rupture. In fact, this provides the mechanism underlying a promising method (load and unload response ratio — LURR) in earthquake forecasting.**

## Introduction

The rupture in heterogeneous brittle media, like materials failure and earthquakes, shows catastrophe transition from globally stable (GS) accumulation of damage to failure. This is called evolution induced catastrophe (EIC). A distinct feature of EIC is its macroscopic uncertainty due to sample-specificity. Its underlying mechanism is a trans-scale sensitivity, i.e. the sensitivity of macroscopic rupture behaviour to the full details of mesoheterogeneity and meso-stress re-distribution. It seems that the very difficulty of rupture and earthquake prediction results from this issue. A scientific strategy to deal with such a complex problem is to explore its universality. We report here that EIC exhibits critical sensitivity and fluctuations near the catastrophe transition point. Such common features can be adopted as rupture precursors and might provide some clues to rupture prediction.

The model we used in our simulation is a coupled pattern mapping. The nonlinear mesoscopic dynamics is iteration of mappings: the evolution of damage pattern is governed by coupled stress and mesoheterogeneity patterns, and the stress pattern is determined by damage pattern according to stress re-distribution (SRD) model. External loading is assumed to be quasi-statically increasing nominal stress  $\sigma_0$ . Then, the damage fraction  $p$  increases from  $p=0$  to  $p=1$ .

Fig. 1 shows the released energy in a mapping  $\Delta E$  versus the damage fraction  $p$ . We can see a catastrophe transition at threshold  $p_c$  (or  $\sigma_{0f}$ ), below  $p_c$  the evolution keeps in a globally stable

(GS) mode and beyond  $p_c$  it falls into the mode of self-sustained catastrophic failure. Further, the evolution can be distinguished into four regimes: (A) an initial regime,  $p \ll p_c$ ; (B) near transition point regime,  $p \leq p_c$ ; (C) an acceleration regime,  $p \geq p_c$ , where the released energy increases rapidly; (D) the main rupture regime  $p \gg p_c$ , where the most energy will be released and the system tends to failure ( $p=1$ ) eventually. The rupture prediction concerns the catastrophe transition at  $p_c$  and the main rupture (regime (D)).

Firstly, there is a universality that the statistics of released energy  $\Delta E$  near the transition point follow a power law in about two decades as shown in Fig. 2. Though the exponents are not universal, the power law does suggest a dynamical criticality related to the catastrophe transition. Therefore, the regime (B) can be called a critical region. The criticality is inherently correlated to an inverse cascade at various scales triggered by minor damage.

Now we introduce a concept called critical sensitivity, which means that some macroscopic behavior of a system shows high sensitivity in the critical region. In order to measure the sensitivity to external load, we define

$$S = \frac{\ddot{A} E'}{\ddot{A} \sigma'_o} / \frac{\ddot{A} E}{\ddot{A} \sigma_o} \quad (1)$$

where  $\Delta E'$  and  $\Delta E$  are the released energy in a mapping induced by increment of nominal stress  $\Delta \sigma'_o$  and  $\Delta \sigma_o$  respectively, and  $\Delta \sigma'_o = \Delta \sigma_o + \alpha \overline{\sigma}_c$ ,  $\overline{\sigma}_c$  is the average strength of units and  $\alpha$  is a small parameter.

Another kind of sensitivity near the transition point is above-mentioned trans-scale sensitivity, which can be detected by minor stochastic damage. We define

$$S^* = \frac{\Delta E^* / \Delta n}{\Delta E} \quad (2)$$

where  $\Delta E$  and  $\Delta E^*$  are the released energy without and with stochastic damage  $\Delta n$ , respectively, under identical loading process.  $S$  and  $S^*$  are shown in Figs. 3(a) and (b) respectively. It is found that  $S$  and  $S^*$  increase rapidly from their initial value of order 1 to high value as a system is approaching to the transition point.

This is a common feature of catastrophe transition, which has been testified by an ensemble statistics based on various SRD models.

Now we examine the fluctuations of stress and damage at the critical point  $\sigma_o = \sigma_{of}$ . Fig. 4 shows that the fluctuations increase strongly at  $\sigma_o = \sigma_{of}$  and become much higher than those in GS regime. Furthermore, the fluctuations of coarse-grained average stress and damage show macroscopic homogeneity in GS regime and high macroscopic inhomogeneity beyond the catastrophe transition. It means a symmetry broken transition at catastrophe transition point, approximately.

Then, how to apply the critical sensitivity and critical fluctuations to rupture prediction? The sensitivity beyond a warning level may be used as a precursor to catastrophe transition. In fact, the critical sensitivity to external load might be the underlying mechanism of earthquake forecasting method called load and unload response ratio (LURR). In addition, the stress fluctuations higher than a warning level and the symmetry broken transition in fluctuations of coarse-grained average stress and damage may be viewed as an immediate precursor of main rupture.

## Figures

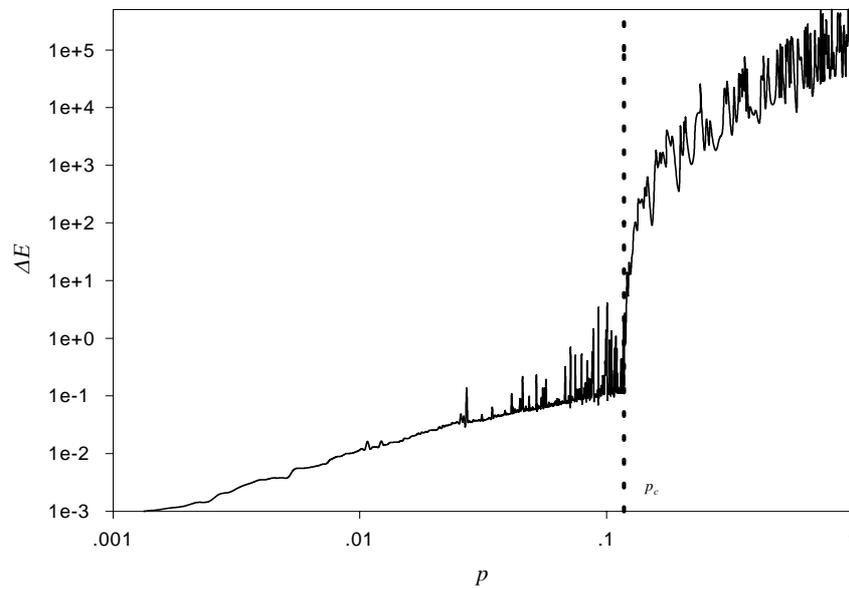


Figure 1: Released energy  $\Delta E$  versus damage fraction  $p$  in damage evolution for cluster mean field (CMF) model. In this system,  $N=10000$ . The process shows evolution induced catastrophe (EIC), i.e. a transition from globally stable (GS) phase to catastrophic failure at transition point  $p_c$ . Other models show the similar EIC behavior.

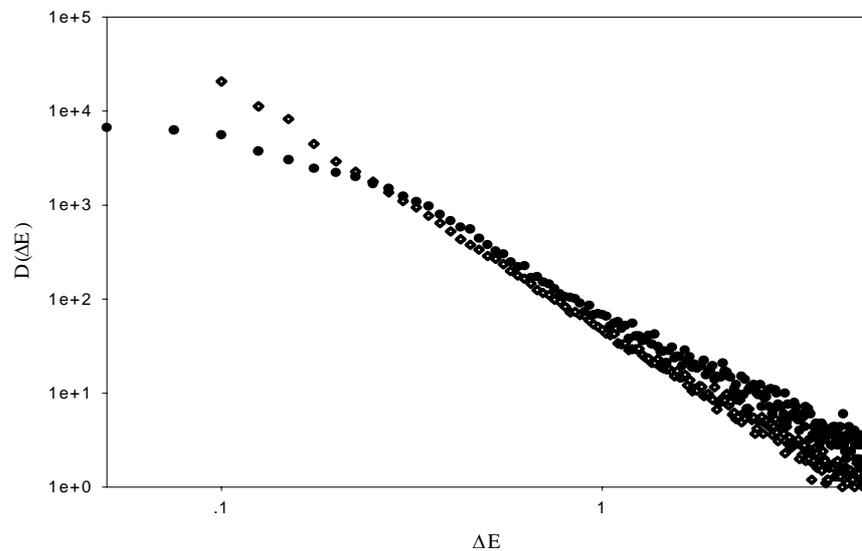


Figure 2: Statistics of released energy  $D(\Delta E)$ , the data are averaging of 2000 samples.  $D(\Delta E)$  shows power law spanning 2 orders, indicating criticality. ●: local mean stress concentration (LMSC) model,  $N=40000$ ,  $\delta=5$ ; ◇: CMF model,  $N=20000$ .

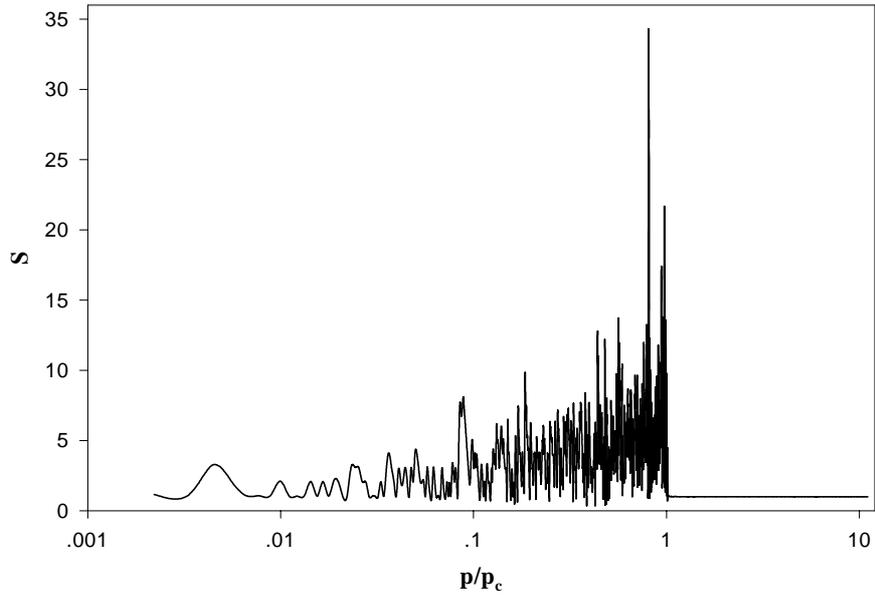


Figure 3(a): Critical sensitivity  $S$  versus  $p/p_c$ , with  $\alpha=0.01$  for CMF model, The sensitivity increases significantly near the transition point.

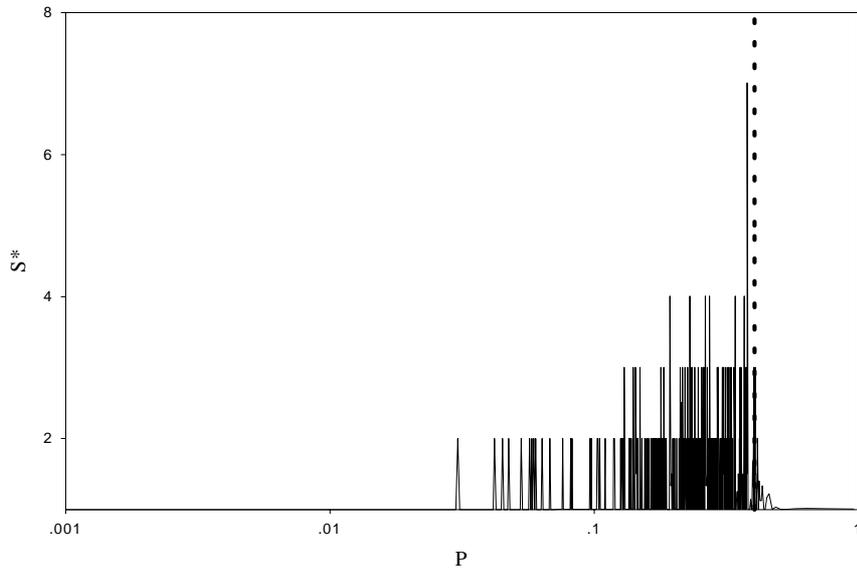


Figure 3(b): Critical sensitivity  $S^*$  versus  $p/p_c$ , for  $N=2000$ ,  $m_c=2$ ,  $\Delta n=1$ , globally mean field (GMF) model The sensitivity increases significantly near the transition point.

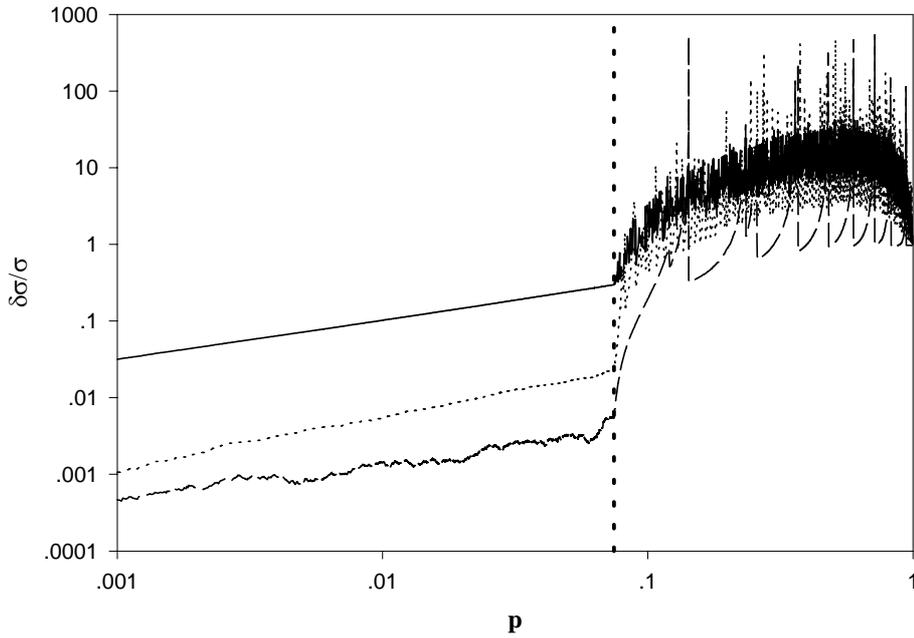


Figure 4(a): The relative deviation of the fluctuations of coarse-grained averaging stress  $[\delta\sigma/\sigma]_c$  versus  $p$  for  $N=65536$ ,  $m_c=2$ , CMF model with coarse-grained scale  $C=1, 256, 4096$ .  
 \_\_\_\_\_  $C = 1$ , .....  $C=256$ , - - - -  $C=4096$

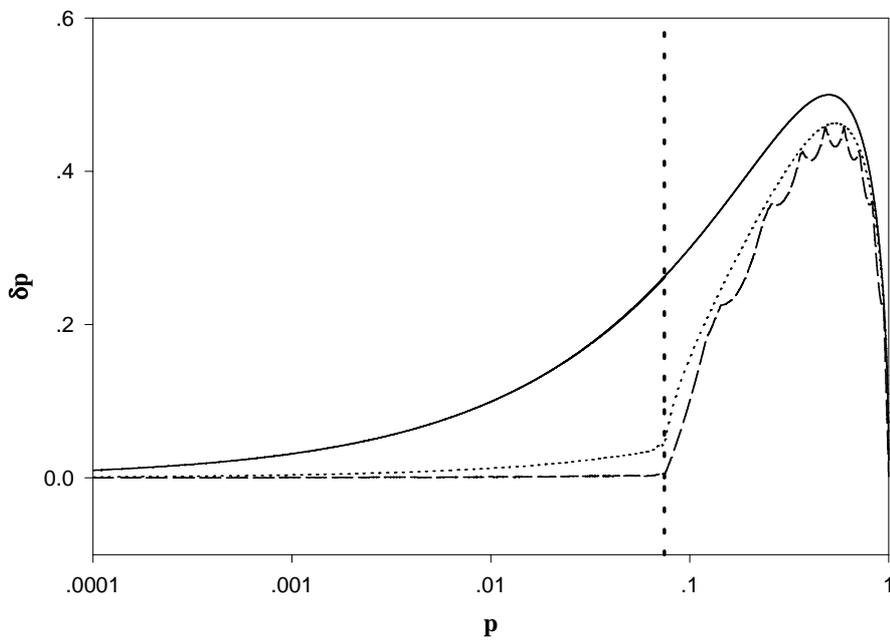


Figure 4(b): The relative deviation of the fluctuations of coarse-grained averaging damage field  $[\delta p]_c$  versus  $p$  for  $N=65536$ ,  $m_c=2$ , CMF model and coarse-grained scale  $C=1, 256, 4096$ .  
 \_\_\_\_\_  $C = 1$ , .....  $C=256$ , - - - -  $C=4096$

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