

# An Optimistic interpretation of friction law from thermal-mechanical coupling in shear deformation of viscoelastic material

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## Abstract

**A thermal-mechanical model of shear deformation of a viscoelastic material as a substitute for frictional resistance is presented. We consider shear deformation of one-dimensional layer composed of a Maxwell viscoelastic material under a constant velocity  $U$  and constant temperature  $T_w$  at the boundary. The strain rate due to viscous deformation depends both on temperature and and shear stress. The temperature inside the layer evolves owing to the competition between frictional heating and conductive cooling. Our results show (i) that the sign of  $d\sigma^{ss}/dU$ , where  $\sigma^{ss}$  is shear stress at the steady state, changes from positive to negative as  $U$  increases, and (ii) that the threshold velocity above which the sign of  $d\sigma^{ss}/dU$  is negative increases with increasing  $T_w$ . These results imply that the downdip limit of seismogenic zones may be marked by the transition in the sign of  $d\sigma^{ss}/dU$  due to temperature rise with depth.**

## Introduction

It is commonly acknowledged that the reason why earthquakes occur only within the uppermost part of the Earth, except for deep-focus earthquakes in the subducting slab, is that ambient conditions, such as temperature, restrict the occurrence of slip instability to within the shallow portion of the lithosphere. To study the influence of temperature on slip behavior is, therefore, crucial to the understanding of the locations and/or depth ranges of seismogenic zones. In this paper, we develop a simple physical model describing slip velocity and stress to study how temperature affects slip behavior.

## Model description

Shear deformation of a Maxwell viscoelastic material, as shown in Figure 1, with an infinite Prandtl number in a layer of half-width  $D$ , is considered. The  $z$ -axis is chosen to run across the sheared layer, and the center and outer boundary are chosen to be  $z = 0$  and  $z = D$ , respectively. We employed a one-dimensional model, i.e. all variables depend only on  $z$  and time  $t$ . The temperature  $T$  is fixed to be  $T = T_w$  at  $z = D$ , while an adiabatic condition is employed at  $z = 0$ . The material is assumed to move in the  $x$ -direction with a constant velocity  $U$  at the outer boundary  $z = D$ , while at the center  $z = 0$  there is no motion. In the present model, the only nonzero elements of stress and strain rate tensors are  $\sigma_{xz}(= \sigma_{zx})$  and  $\dot{\epsilon}_{xz}(= \dot{\epsilon}_{zx})$ , hereafter denoted by  $\sigma$  and  $\dot{\epsilon}$ , respectively.

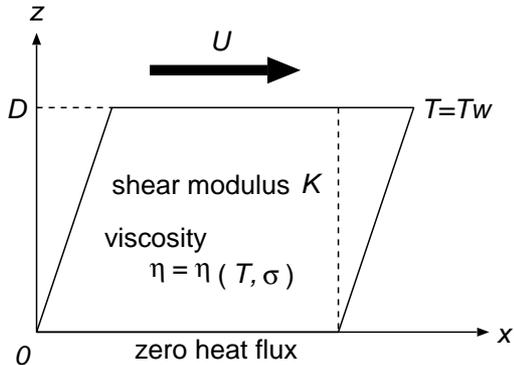


Figure 1: Illustration of model used in this study.

The numerical model is developed in a non-dimensional form. The conversion into non-dimensional quantities is carried out with a length scale of  $D$ , time scale of  $D^2/\kappa$  ( $\kappa$  is thermal diffusivity), stress scale of  $K$  (shear modulus), and temperature scale of  $K/\rho C_p$  ( $\rho C_p$  is volumetric heat capacity). The non-dimensional forms of the basic equations are,

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2} + \sigma \dot{\epsilon}_v, \quad (1)$$

$$\frac{d\sigma}{dt} = U - \int_0^1 \dot{\epsilon}_v(\sigma, T(z)) dz, \quad (2)$$

$$\dot{\epsilon}_v = C_n \sigma^n \exp\left(-\frac{h_n}{T}\right), \quad (3)$$

$$T(z=1) = T_w, \quad (4)$$

$$\frac{\partial T}{\partial z}(z=0) = 0, \quad (5)$$

where  $t$  is elapsed time,  $\dot{\epsilon}_v$  is strain rate due to inelastic (viscous) deformation, and  $C_n$ ,  $n$ , and  $h_n$  are parameters. In the calculations presented below, we chose  $C_n = 4.69 \times 10^{38}$ ,  $n = 3.5$ , and  $h_n = 1.949$ , which give a strain rate close to that associated with dislocation creep of dry olivine ([1], [2]).

The basic equations are discretized into a finite difference grid based on the control volume method [3]. The computational domain was divided uniformly into 2000 mesh divisions. We carried out both steady-state calculations and time-dependent calculations. When the purpose is to obtain a steady-state solution, we solved the above equations, after letting  $\partial/\partial t = 0$ , by the standard shooting method. When the purpose is to obtain a time-dependent solution, we chose the time stepping  $\delta t$  to satisfy the Courant condition, unless unstable slip occurs. When unstable slip occurs, we chose  $\delta t$  short enough for the relative stress drop  $|\delta\sigma/\sigma|$  during  $\delta t$  to be less than  $O(10^{-2})$ . The time derivative in the energy equation (1) is discretized by a first-order explicit scheme. The time integration of the equation of stress change (2) is carried out by fourth-order Runge-Kutta method. The reliability of this numerical code was verified in Kameyama et al. (1999 [2]).

## Results

We carried out steady-state calculations for various values of  $T_w$  and  $U$ . (In what follows, we will only discuss the results of steady-state calculations, although time-dependent solutions will be discussed in the actual presentation.) We show in Figure 2 the distributions of (a) temperature  $T$  and (b) strain rate  $\dot{\epsilon}$  at the steady state obtained in Case A where  $T_w = 0.024$  and  $U = 5 \times 10^{-2}$ . As can be seen from the figure, the steady state is characterized by

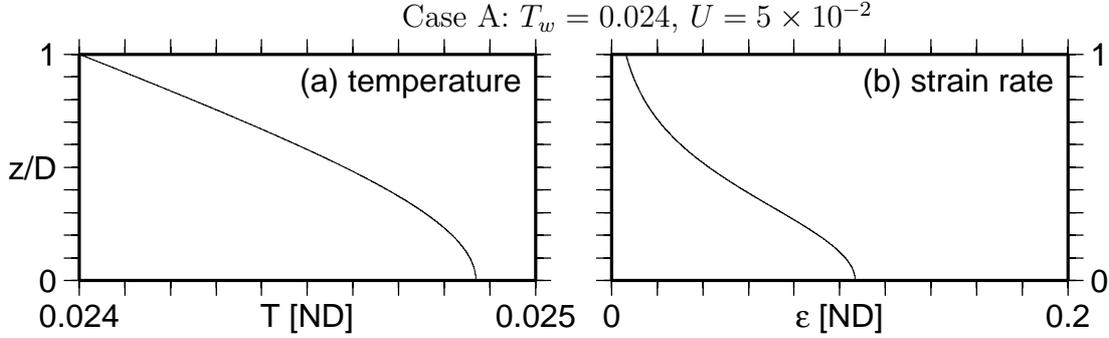


Figure 2: Plots of distributions of (a) temperature  $T$  and (b) strain rate  $\dot{\epsilon}$  at the steady state of Case A. Parameter values are  $T_w=0.024$  and  $U = 5 \times 10^{-2}$ .

maximum  $T$  and  $\dot{\epsilon}$  near  $z = 0$ . These distributions are the results from the competition between frictional heating and conductive cooling. Frictional heating raises the temperature in the entire layer. On the other hand, conductive cooling from the outer boundary largely suppresses the temperature rise near the outer boundary  $z = D$  while it does not significantly suppress the temperature rise near  $z = 0$ . Because temperature is highest, strain rate is also highest near  $z = 0$ .

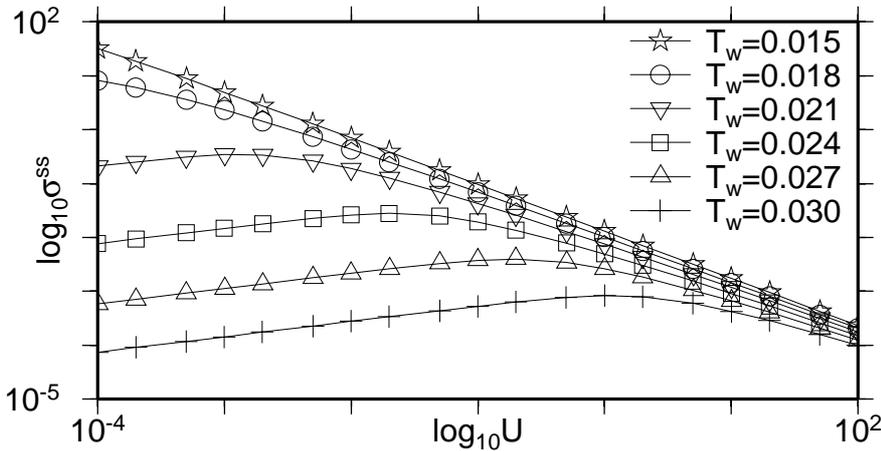


Figure 3: Plots of shear stress at the steady state  $\sigma^{ss}$  against  $U$  for the various values of  $T_w$ .

In Figure 3, we show the plots of steady-state stress  $\sigma^{ss}$  against the shearing velocity  $U$  for various values of  $T_w$ . The figure shows (i) that the sign of  $d\sigma^{ss}/dU$  changes from positive to negative as  $U$  increases, and (ii) that the velocity  $U_{th}$  where the sign of  $d\sigma^{ss}/dU$  changes

becomes larger as  $T_w$  is higher. In other words, the stability of steady-state slip changes according to the changes in  $T_w$  and  $U$ .

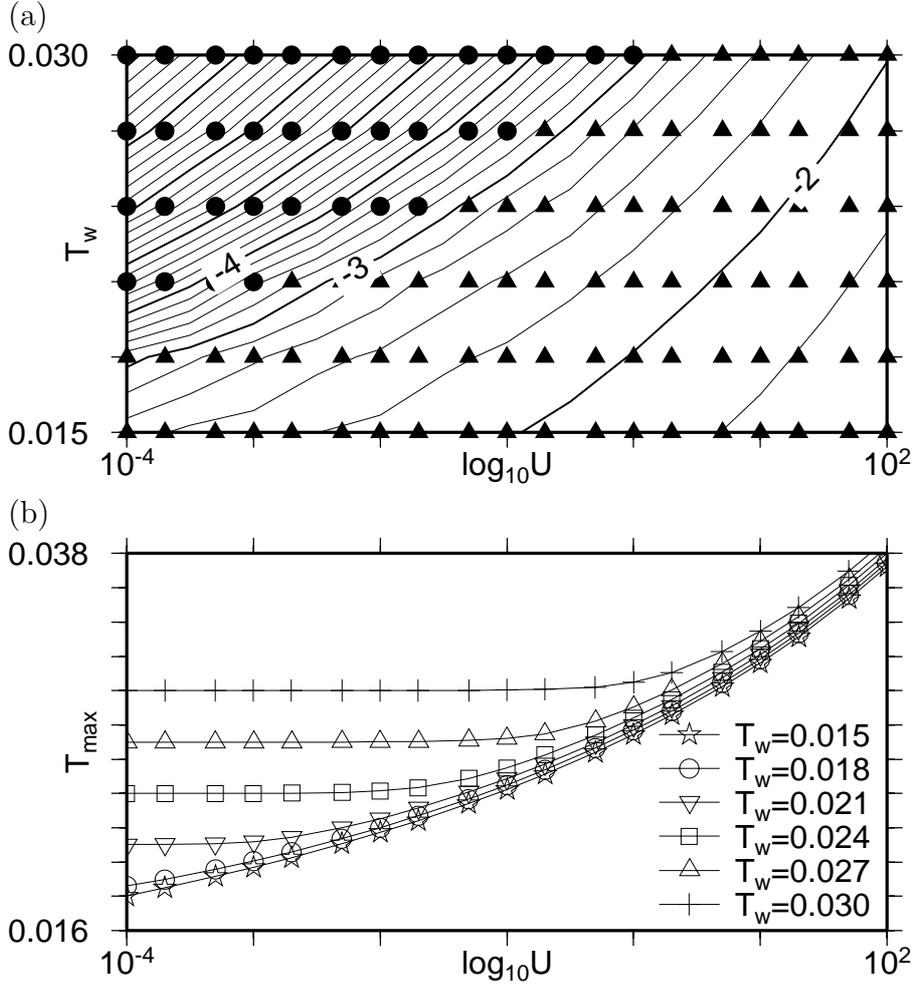


Figure 4: (a) Diagram for the sign of  $d\sigma^{ss}/dU$  for various values of  $U$  and  $T_w$ , and (b) plots of maximum temperature  $T_{max}$  within the computed layer at against  $U$  for the various values of  $T_w$ . In (a), solid circles indicate positive  $d\sigma^{ss}/dU$ , while solid triangles indicate negative  $d\sigma^{ss}/dU$ . The solid contours indicate the logarithm of the rate of frictional heating  $\Phi \equiv \sigma^{ss} U$ .

To see why the sign of  $d\sigma^{ss}/dU$  changes as  $U$  increases, we show in Figure 4 (a) the sign of  $d\sigma^{ss}/dU$  for various values of  $U$  and  $T_w$ , and (b) the plots of maximum temperature  $T_{max}$  against  $U$ , for the various values of  $T_w$ . In Figure 4a, solid circles indicate positive  $d\sigma^{ss}/dU$ , while solid triangles indicate negative  $d\sigma^{ss}/dU$ . We also show with the contours in Figure 4a the logarithm of the amount of frictional heating  $\Phi \equiv U\sigma^{ss}$ . Figure 4 shows that the reason why  $d\sigma^{ss}/dU$  switches from positive to negative as  $U$  increases is because  $\Phi$  becomes sufficiently large and  $T_{max}$  significantly deviates from  $T_w$ . When  $U$  is significantly low, a slight increase in  $U$  does not sufficiently raise the temperature in the layer, because the amount of frictional heating does not significantly increase. The effective viscosity is almost unchanged and, hence,  $\sigma^{ss}$  becomes higher in proportion to the increase in  $U$ . When  $U$  is high, in contrast, even a slight increase in  $U$  sufficiently increases  $\Phi$  and raises the temperature in the layer. The effect of the decrease in the effective viscosity overcomes the increase in  $\sigma^{ss}$  due to the increase in  $U$ , and  $\sigma^{ss}$  decreases as  $U$  increases.

Figure 4 also shows that the reason why  $U_{th}$  becomes larger when  $T_w$  is higher is because a larger amount of frictional heating is required to reduce  $d\sigma^{ss}/dU$  for higher  $T_w$ . As can be seen from the equation (4), the degree of the decrease in the effective viscosity due to temperature increase is smaller when  $T_w$  is higher. Hence, a larger amount of frictional heating or, in other words, a higher  $U$  is required to sufficiently raise the temperature in the layer and to change the sign of  $d\sigma^{ss}/dU$ . By linear stability analysis of the steady state (Kameyama et al., in preparation), we found that the amount of frictional heating  $\Phi_c$  for  $d\sigma^{ss}/dU$  to be negative is approximately equal to  $2T_w^2/h$ , which is consistent with  $\Phi_c$  shown in Figure 4a.

## Concluding remarks

We develop a simple physical model describing slip velocity and stress based on thermal-mechanical coupling in shear deformation of a viscoelastic material. We demonstrate (i) that the sign of  $d\sigma^{ss}/dU$  changes from positive to negative as  $U$  increases, and (ii) that the velocity  $U_{th}$  where the sign of  $d\sigma^{ss}/dU$  changes becomes larger as  $T_w$  is higher. In other words, for a given slip rate, the steady-state slip is prone to be unstable for lower temperature, while is likely to be stable for higher temperature. These results imply that the downdip limit of seismogenic zones may be marked by the transition in the sign of  $d\sigma^{ss}/dU$  due to temperature rise with depth.

## References

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