

Large-scale Parallel Wave Propagation Analysis (Explicit dynamic analysis using non-reflecting boundary)

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Abstract

GeoFEM is solid earth simulator software, which is under development to be used by STA's super parallel computer, "Earth Simulator (GS40)". GeoFEM has already been used to analyze large-scale static linear problems and wave propagation problems up to 10^8 degrees of freedom (DOF) and demonstrated high performance computing ability by parallel processing. In the present study, we focus on the wave propagation analysis of GeoFEM. The report reviews formulation and parallel benchmark result briefly, then describes installations of viscous boundary and real-time visualization system (RVSLIB) to realize more efficient simulation. According to these extensions, we can apply GeoFEM to more realistic seismic problem.

Introduction

GeoFEM[1] is parallel FEM code developed to simulate solid earth phenomena. In the present study, to extend usability of GeoFEM wave propagation analysis function, we install viscous boundary and real-time visualization system (RVSLIB) to GeoFEM. The former is to realize non-reflecting boundary, and the latter is to handle huge output data from explicit dynamic analysis.

Method for wave propagation analysis in GeoFEM

Applying the FEM to a discrete-space system, the equation of motion is expressed by mass matrix $[M]$, dumping matrix $[C]$, internal force vector $\{p\}^{(n)}$, external force vector $\{f\}^{(n)}$, displacement vector $\{u\}^{(n)}$, and its first and second order time derivation, velocity and acceleration vector, $\{\dot{u}\}^{(n)}$, $\{\ddot{u}\}^{(n)}$ respectively (1). Here suffix $^{(n)}$ means at time t .

$$[M]\{\ddot{u}\}^{(n)} + [C]\{\dot{u}\}^{(n)} + \{p\}^{(n)} = \{f\}^{(n)} \quad (1)$$

Equation of motion (1) can be rewrote by using central difference equation as following[2].

$$\begin{aligned} \{u\}^{(n+1)} = & \left[[M] + \frac{\Delta t}{2}[C] \right]^{-1} \\ & \left\{ \Delta t^2 \left[\{f\}^{(n)} - \{p\}^{(n)} \right] + 2[M]\{u\}^{(n)} - \left[[M] - \frac{\Delta t}{2}[C] \right] \{u\}^{(n-1)} \right\} \end{aligned} \quad (2)$$

In order to solve equation (2) explicitly, we transform the mass matrix $[M]$ and dumping matrix $[C]$ to diagonal form. Under these assumptions, equation (2) can be rewritten as a scalar equation, composed of i -th vector components denoted by i and diagonal matrix components denoted by ii .

$$u_i^{(n+1)} = \frac{\Delta t^2(f_i^{(n)} - p_i^{(n)}) + 2M_{ii}u_i^{(n)} - (M_{ii} - \frac{\Delta t}{2}C_{ii})u_i^{(n-1)}}{M_{ii} + \frac{\Delta t}{2}C_{ii}} \quad (3)$$

Equation (3) is conditionally stable, since it is limited by the time increment Δt . This restriction is known as the ‘‘Courant condition’’.

In GeoFEM, the internal force is evaluated using equation (4) assuming a linear problem. Here, $[K]$ is the stiffness matrix of whole system, same as a linear static problem.

$$\{p\}^{(n)} = [K]\{u\}^{(n)} \quad (4)$$

The method can be used to analyze linear wave propagation problems with high-speed computation.

Parallel performance evaluation in large-scaled problem

Simple benchmark was executed to evaluate the performance of the wave propagation analysis for a large-scale parallel problem[3]. The parallel computation was executed using in Hitachi SR2201 of the University of Tokyo. Figure 1 shows the elapsed time measured by benchmark analyses varying scale of analysis and PE count. The maximum DOF case gives a total node count of $330 \times 330 \times 330 = 35,937,000$ or $107,811,000$ DOF, that is, approximately 10^8 DOF.

Figure 2 shows the scalability of S_n by assuming the parallel efficiency is equal to the rate of CPU usage: W_n . The figure shows nice scalability even though massive parallel computation. For the largest scale problem (10^8 DOF, 1000PE) the rate of CPU usage was calculated $W_n=98.23\%$ and speed-up $S_n=982$.

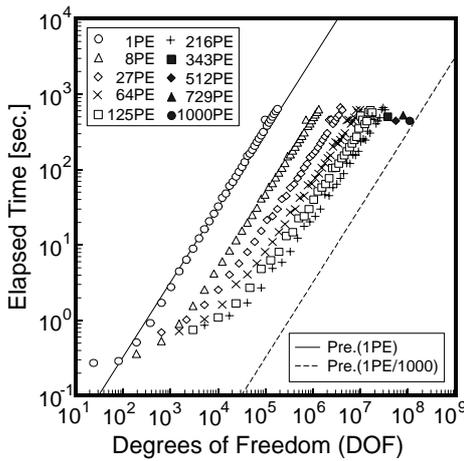


Figure 1: Elapsed Time by Cubic Model

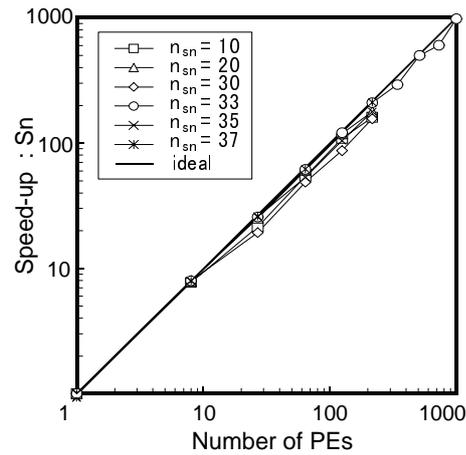


Figure 2: Scalability

Non-reflecting boundary using viscous damping

Seismic wave propagation analysis using FEM is required non-reflecting boundary for side or bottom of the model, because the area of mesh is finite. The viscous damping boundary method[4] is one of the most efficient, due to the less of computation cost to realize several proposed non-reflecting boundary method. The method is to replace the semi-infinite wave propagation phenomena by the boundary viscous damping model approximately. Assuming a passage of the wave at boundaries, the model gives proper stress components on the boundary. At the three dimensional stress field, the boundary stress components was written by the following equations, when incident wave consists of primary or secondary body waves and its angle is expressed by θ from z-axis as figure 3.

$$\sigma_{zz} = a\rho V_p \dot{u}_z \quad (\text{Primary wave}) \quad (5)$$

$$\tau_{zx} = b\rho V_s \dot{u}_x \quad (\text{Secondary wave}) \quad (6)$$

$$\tau_{yz} = b\rho V_s \dot{u}_y \quad (\text{Secondary wave}) \quad (7)$$

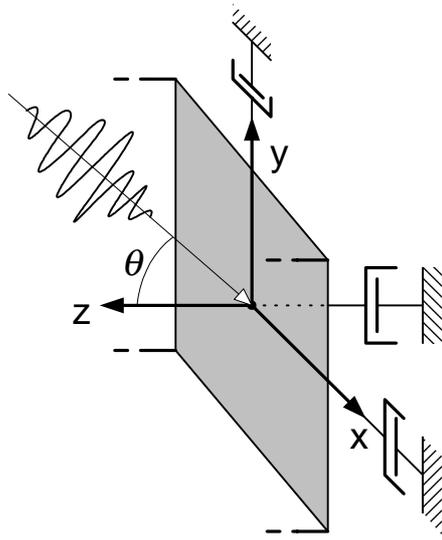


Figure 3: Viscous Boundary Concept

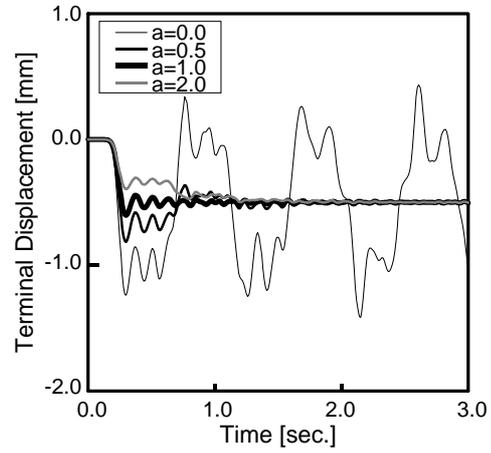


Figure 4: Effect of Viscous damping

- ρ : density
- V_p : P-wave velocity, $V_p = \sqrt{\frac{(1-\nu)E}{(1-2\nu)(1+\nu)\rho}}$, (E : modulus of elasticity)
- V_s : S-wave velocity, $V_s = \sqrt{\frac{E}{2(1+\nu)\rho}}$, (ν : Poisson's ratio)
- $\dot{u}_z, \dot{u}_x, \dot{u}_y$: velocity of z,x,y-direction respectively

Where a, b means non-dimensional parameters and the value of 1.0 is the most effective to absorb the reflecting energy as the incident angle θ is not large[4]. In GeoFEM, following lumped damping term is added to boundaries to produce boundary stress.

$$c_{ii} = \int a\rho V_p ds \quad \text{Primary wave, } z \text{ direction} \quad (8)$$

$$c_{ii} = \int b\rho V_s ds \quad \text{Secondary wave, } x \text{ or } y \text{ direction} \quad (9)$$

Figure 4 shows verification result of viscous boundary by uni-axial lateral vibration problem of a beam. Here $a = 0.0$ means free and $a = \infty$ means fixed terminal condition. As the viscous boundary theory, $a = 1.0$ case controls wave reflection. Though the effectiveness of this method depends on the incident angle in three-dimensional case and could not control the surface wave.

Real-time visualization system: RVSLIB

The wave propagation analysis using explicit dynamic response method has to deal with huge output data, because the scale of the analysis is relatively large due to small computation cost, and it requires enormous time stepping due to small time increment. The real-time visualization system (RVSLIB)[5] which is a commercial product of NEC Corporation, was installed in GeoFEM to solve this kind of problem.

RVSLIB adapts client server system. RVSLIB/server library is linked to GeoFEM/analyzer module. When the GeoFEM/analyzer is executed, RVSLIB/server generate the visualization images simultaneously. These images can display by the RVSLIB/client in real-time though the network, furthermore, RVSLIB/client can control analysis by client-server communication. The method is one of the solution for huge data transfer problem in separate execution of analysis and post-processing, and it improves the usability by using tracking or steering option especially large-scale analyses.

Conclusion

The present study, we focused on the wave propagation analysis of the GeoFEM system. The formulation, parallel performance and installation of non-reflecting boundary and RVSLIB have been described. In conclusion, due to the present extension, the GeoFEM wave propagation analysis function becomes applicable for realistic seismic problem to simulate solid earth phenomena.

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References

- [1] <http://geofem.tokyo.ri.t.u-tokyo.ac.jp>
- [2] D.R.J. Owen, E.Hinton, *Finite Elements in Plasticity*, Pineridge Press, 1980.
- [3] K.Garatani, H.Nakamura, H.Okuda and G.Yagawa, Large-scale Parallel Wave Propagation Analysis by GeoFEM, HPCN Europe 2000; LNCS-1823, 445-543. 2000.
- [4] J. Lysmer, R. L. Kuhlemeyer Finite dynamic model for infinite media, Proc. ASCE, Vol. 95, No.EM4, 1969.
- [5] http://www.sw.nec.co.jp/APSOFT/SX/rvslib_e/