

Finite-element simulation of seismic wave propagation with a voxel grid

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Abstract

The finite element method (FEM) can accurately calculate seismic ground motions for complex topography, since the traction-free conditions are already been cast in its formulation. However, FEM usually requires large memory and takes a long computation time. We introduce a grid consisting of voxels (rectangular prisms) and take isotropy into the explicit formula of the dynamic matrix equation for overcoming these defects. Compared with the finite difference method (FDM), the voxel FEM requires a similar amount of memory and takes only 1.4 times longer computation time. We can generate a voxel mesh much faster than a popular tetrahedron mesh. We first calculate ground motions and static displacements due to a point source in a halfspace or three-layer structure. We then compare them with results of the reflectivity method and theoretical solutions for verification. The voxel FEM achieves good agreement. We also demonstrate an inherent advantage of FEM at the free surface.

Formulation

In seismology, the finite difference method (FDM) has been popular for many years rather than the finite element method (FEM), though FEM have some inherent advantages over FDM. For example, the FEM solution satisfies the free-surface condition by nature, since the basic equation of FEM is derived based on the traction-free condition at the outer boundary of the medium. As a reason for the popularity of FDM, it can be thought that FDM usually requires much less computer memory and a shorter computation time than FEM. For overcoming these defects of FEM, we will here introduce a grid consisting of voxels (rectangular prisms) and derive an explicit formula of the dynamic matrix equation assuming isotropic media.

‘Voxel’ is a term in Computer Graphics and means a volume pixel, which is actually a rectangular prism. The generation of the voxel mesh is as easy as in FDM, when we use the simplest linear shape functions with regular intervals (Figure 1). Komatitsch and Tromp (1999)[4] performed the exact diagonalization of the mass matrix using irregular spacing based on the Legendre polynomials, but we choose the regular spacing giving priority to the easy mesh generation. Since the nodal coordinates of the elements can be easily calculated and so do not need to

be memorized for this regular spacing, memory requirement is significantly reduced compared to the case where the coordinates are memorized.

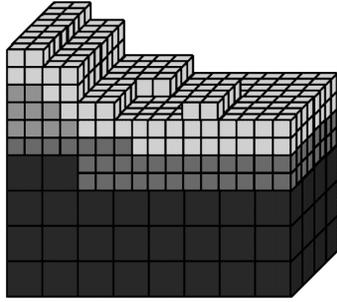


Figure 1: Voxels in a medium.

Secondly we take isotropy into the formulation explicitly. If an element is fully anisotropic, there are 24 independent components in the stiffness matrix. However, according to the symmetry among the isotropic elastic constants and linear shape functions, the number of independent components is reduced to 12. This reduction is also effective for increasing the computational efficiency of FEM.

The voxel FEM has already been implemented for a Beowulf PC cluster with the MPI library.

Numerical Examples

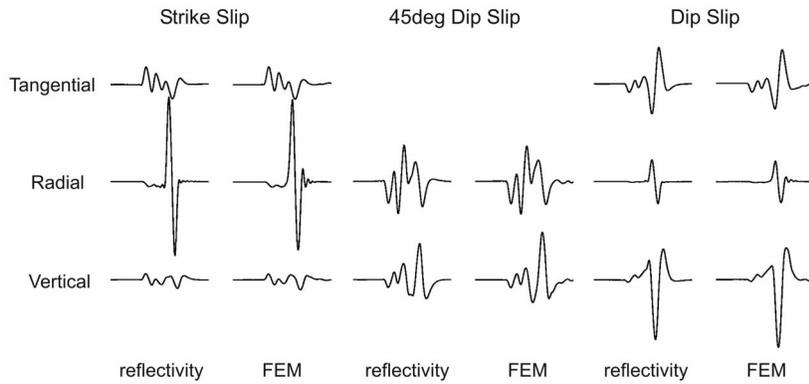


Figure 2: Ground motions in a halfspace (left: reflectivity method, right: FEM).

We calculate ground motions from a point source in the halfspace ($V_P = 4.0$ km/s, $V_S = 2.3$ km/s, $\rho = 1.8$ g/cm³) or three-layer structure of Graves (1996)[2] using the voxel FEM. We then compare them with results of the reflectivity method (Kohketsu, 1985 [3]) for verification. The voxel FEM achieves good agreement shown in Figures 2 for three kinds of point sources, *i.e.*, a strike slip, 45-degree dip slip and vertical dip slip at a depth of 2.5 km in the halfspace with a cosine time function 1 s long. It takes 1.4 hour for the voxel FEM to complete the 20 s (800 steps;

0.025 s interval) time history of a $30 \times 30 \times 10$ km medium (4,608,000 elements; 125 m interval) on a 1.7 GHz Pentium 4 with 1 GB RIMM memory, while FDM of Furumura *et al.* (2000)[1] spends 1.0 hours for the same configuration. The voxel FEM requires a similar amount of memory to that of FDM.

The next verification is carried out for a strike slip source at a depth of 1.6km in a horizontally layered structure. This three-layer model was also used by Graves (1996)[2]. The FEM code occupies 675MB memory for 12,800,000 elements, and again achieves good agreement as shown in Figure 3.

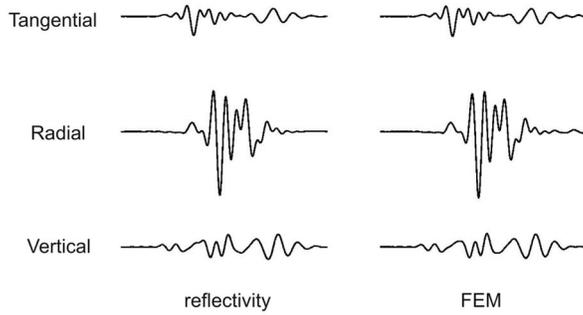


Figure 3: Ground motions in a layered structure (left: reflectivity method, right: FEM).

Wald and Graves (2001)[6] demonstrated that FDM could calculate even static displacements if the computation continues for a sufficiently long time. This demonstration is confirmed using the voxel FEM in the same halfspace for the previous tests. Figure 4 favorably compares the displacements by FEM with the theoretical solutions by Okada (1985)[5], though GMT and micro AVS give different impressions for similar results.

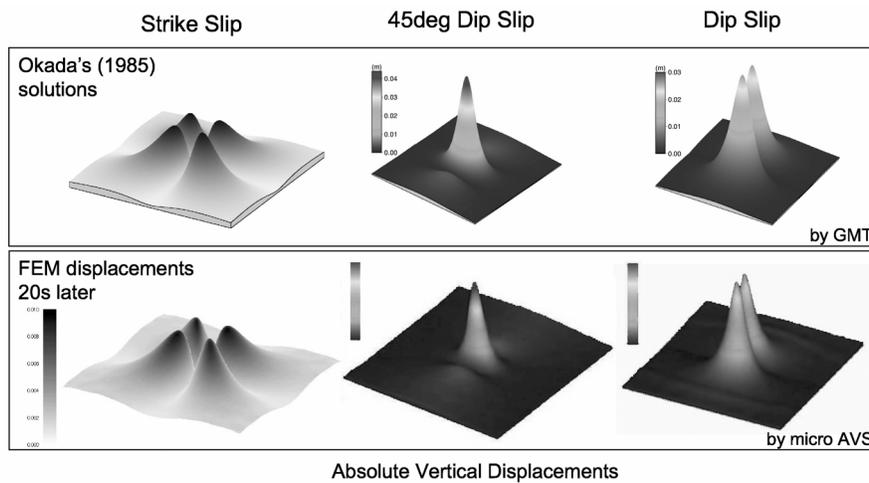


Figure 4: Static displacements due to a strike slip (upper: Okada's solution, lower: FEM).

One of the greatest advantages of FEM is that the traction-free condition has already been cast in the formulation. No special treatment is needed for the free

surface. In order to confirm this advantage, another test of Graves (1996)[2] is carried out in the halfspace. The dashed FK seismograms in Figure 5 should be correct. Although the FDM results fail to agree with them, the results by the voxel FEM achieve good agreement.

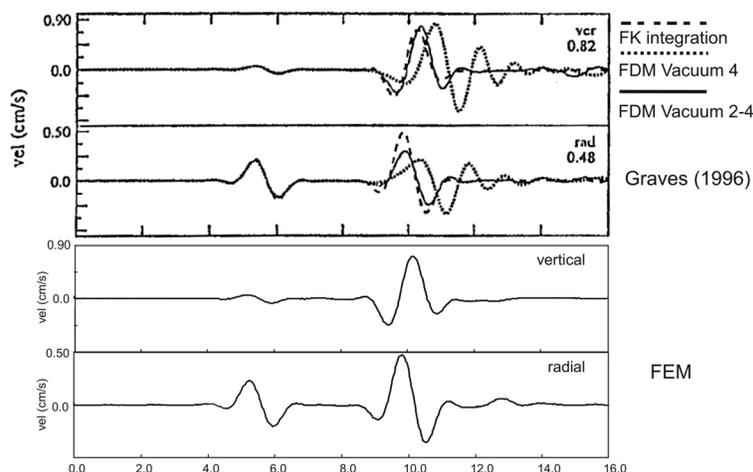


Figure 5: Comparison of seismograms calculated with the FK technique, FDM using the modified vacuum formulations, and the voxel FEM.

Acknowledgments

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