Energy Densities of Diffuse Fields in Seismology: The Case of a Layered Medium

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Both the microtremors and coda of earthquake motions are produced by multiple scattering due to the heterogeneity of the Earth. They are usually referred to as diffuse wave fields and it is accepted that its intensities are governed by diffusion-like equations.

Within an inhomogeneous, anisotropic elastic medium in which an uncorrelated set of random forces is acting (equivalently, it is subjected to diffuse seismic illumination), the average cross correlation of the motions at pairs of given receivers, in the frequency domain, is proportional to the imaginary part of Green function for such locations. For a single recording point, the average autocorrelation measures energy density and it is proportional to the imaginary part of Green function, when both loading and receiver are the same.

It is remarkable that average correlations or autocorrelations of stochastic measurements lead to intrinsic properties of medium. The imaginary part of Green function at the loading point is also proportional to the power injected into the medium by a unit harmonic load at that point. As the load is unit such power will vary if the motions are affected by waves that are reflected back to the source location. Therefore, it represents reflection events. In fact, the average autocorrelations in time domain are called pseudo-reflection seismograms. These properties of diffuse fields can be used to imaging the mechanical and geometrical characteristics of an elastic domain.

To explore such properties we consider a horizontally layered elastic medium overlying an elastic half-space. We assume that observations are at the free surface. Two extreme cases are of interest: The fields can be one- or three-dimensional (1D or 3D) and consequently the Green's function can be 1D or 3D, respectively.

As a working hypothesis we consider that some of those diffuse seismic fields can possibly be established in a horizontally layered medium overlying a half space, we compute the H/V spectral ratio in terms of the imaginary part the Green function tensor components at the surface. Our theory links average energy densities with the Green function, an intrinsic property of the medium. This naturally allows the medium inversion from the surface H/V spectral ratio.

Microtremors are generated close to the Earth surface, have random origin, and are of 3D nature. By linking directional spectral densities with energy density and with the imaginary part of the corresponding Green function components we obtain theoretical expressions for the average H/V spectral ratio, the well known Nakamura's ratio. Our theory does not require using the ellipticity of Rayleigh waves. It includes the contributions of Rayleigh, Love and body waves.

On the other hand, relatively deep earthquakes illuminate the site structure from below and may be considered as composed by plane waves. Based on diffuse field concepts we assume that the average autocorrelations for several earthquakes represent directional energy densities associated to the site. These densities are proportional to the imaginary part of 1D Green function at the free surface. We found that the imaginary part of Green function at the free surface is proportional to the square of the absolute value of the corresponding transfer function for a plane, vertically incident wave with unit amplitude.