

# Construction of Curvelet Solutions to Tsunami Equations from a Multi-Scale Framework

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In recent years wave propagation in strongly heterogeneous media has been a hot topic for the elastic wave equations because of the intense interest arising from both deep- earth seismic tomography and seismic imaging in exploration geophysics. Recent advances have been made in unraveling the multi-scale nature of propagating elastic waves, which are complex because of strong 3-D heterogeneities and anisotropies. This has been made possible by utilizing micro-local techniques on the pseudo-differential operators associated with systems of hyperbolic partial differential equations with strongly variable coefficients. Curvelets, which is a new development of multi-scale transform with a strongly directional character, can be demonstrated to be an exact solution of these types of hyperbolic partial differential equations. There is a great deal of improved computational efficiency in the wave propagation brought about by the use of groups of curvelets or packlets in the elastic wave problem. The solutions to the system of first-order hyperbolic equations governing the shallow-water equations for tsunami waves have multi-scale character because of the coefficients in front of the differential operators , which are associated with strongly laterally heterogeneous nature of the seafloor topography . Therefore we have employed the same curvelet techniques which have been successful for solving systems of elastic wave propagation on the linear tsunami wave equations in the long wavelength regime. We will lay out the mathematical formalism based on Hamiltonian principle using curvelets which are needed to solve the linear shallow water equations with multi-scale bottom topography . They can lead to great speed-ups in the computational time for the travel time of tsunami waves over conventional methods based on classical techniques, such as Greens functions or normal modes, finite-differences and finite-elements. These mulit-scale solutions can be corrected by higher-order terms and second-order corrections are deemed to be sufficient for accurate capturing of caustics from islands. We will also discuss the deployment of a finite-volume adaptive- grid refinement ( AMR ) techniques applied to tsunami wave equations.